ILLUSTRATIONS OF ARCHITECTURAL SHADOWS.

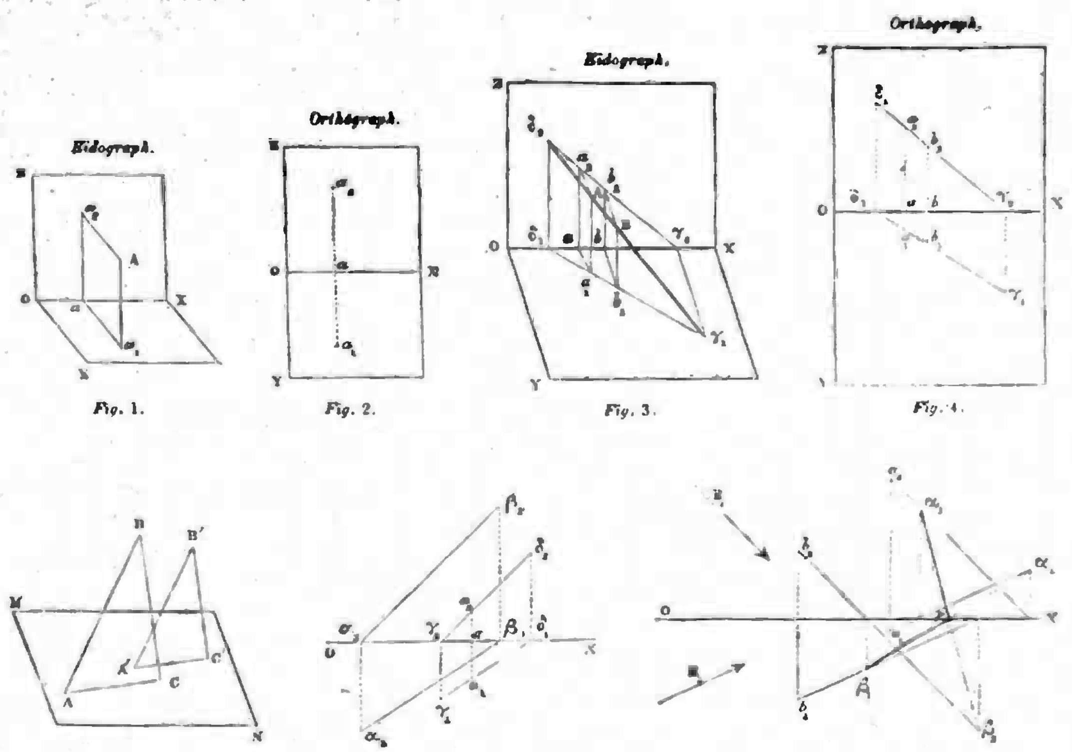

Fig. 1. Fig. 2. Fig. 3. Fig. 4.

Fig. 5. Fig. 6. Fig. 7.

plane XY (*xi.* 38), and hence (*xi.* 6) a_2a is parallel to Aa_1. Similarly it is proved that a_2A is parallel to aa_1; and hence Aa_2aa_1 is a parallelogram. Also it has one right angle a_2aa_1, and is therefore a rectangle (i. 29 and 34).

Let us now conceive the lines Aa_1, Aa_2 to be removed, leaving the lines a_1a, a_2a in their respective positions in the planes XY, XZ. Then let us conceive the plane XY to be revolved about OX till it comes into coincidence with the plane XZ *produced.* The result will be the *orthograph.* Then since Xaa_1 is a right angle, and the relation of Xa to aa_1 is not altered by the revolution about Xa, it will be a right angle in the orthgraph; and a_1aa_2 becomes a single straight line perpendicular to OX.

In this it will be seen at once, that aa_1 is the height of the point A of the eïdograph *above* the plane XY, and that a_1a is the distance of the same point A *before* the elevation XZ. And if we take away the frame-work of the planes XY, XZ, we shall have the ordinary "descriptive representation" of the point A of the eïdograph.

Again, if there be a second point B in the eïdograph, its "descriptive representation" will be b_1bb_2, as in the figured orthograph.

Also, if AB be a given line in the eïdograph, the projections a_1b_1, a_2b_2 constitute its descriptive representations in the orthograph. Figs 3 and 4.

II. Let us now conceive the line AB produced both ways (if necessary) to pierce the plan and elevation planes in γ_1 and δ_2. These are called the *traces* of the line AB. Then two converse problems arise, which it will be necessary to specify.

PROBLEM A.—*Given the projections of the two points in a line, to find the traces of that line.*

ANALYSIS.—Let the eïdographic plane ABb_1a_1 be produced and cut XY, XZ, in $\gamma_1\delta_1$, $\delta_1\delta_2$, and the plane ABb_2a_2 to cut them in $\gamma_1\gamma_2$, $\gamma_2\delta_2$. Then, since both planes pass through AB, their intersection is the line AB produced to meet the planes XY, XZ; or γ_1, δ_2 will be the *traces* of the line AB.

Now since by the definition of orthographic projection, Aa_1 and Bb_1 are perpendicular to XY, the plane $\gamma_1\delta_1\delta_2$ is perpendicular to XY (*xi.* 18). Whence, since the two planes $\gamma_1\delta_1\delta_2$ and XZ are perpendicular to XY, their intersection $\delta_1\delta_2$ is also perpendicular to XY XY (*xi.* 19); and hence (*xi.* 4) $\delta_1\delta_2$ is perpendicular to OX. But a_1, b_1 are given, and hence the intersection δ_1 of a_1b_1 with OX is given; and therefore the vertical trace of the line AB is in the given perpendicular from δ_1. Moreover, the vertical trace of AB is in the given vertical projection a_2b_2 of the line AB. Whence it is their intersection.

In the same way the horizontal trace is shewn to be the intersection of a_1b_1 with $\gamma_2\gamma_1$; and the following simple construction results:—

SYNTHESIS.—Let a_1b_1, a_2b_2 be the horizontal and vertical projections of the line AB upon the *orthograph.* Produce them to meet the ground line OX in δ_1 and γ_2 respectively. Draw the perpendicular $\gamma_2\gamma_1$, $\delta_1\delta_2$ to meet the projections a_1b_1, a_2b_2 (produced if necessary) in γ_1, δ_2. These will be the traces required.

The demonstration is obvious from the analysis and need not be put down here.

PROB. B.—*Given the traces of a line to find its projections.*

The following construction results from a slight variation of the preceding analysis.

SYNTHESIS.—Let γ_1, δ_2 be the traces on the orthograph of the given line. Draw $\gamma_1\gamma_2$, $\delta_2\delta_1$, perpendicular to the ground line OX; and join $\gamma_1\delta_1$, $\gamma_2\delta_2$. These are the projections of the entire of the *visible part* of the line; that is, of the part which lies in the "region of space" inclosed by the two planes which are opposite to the spectator.

III. *The projections of parallel lines are themselves parallel, whatever be the plane of projection, and whatever be the direction of the projectors.*

Let MN be the plane of projection, AB, A′B′ two parallel lines meeting MN in A,A′. Take any points, B,B′ in these lines, and draw BC, B′C′ parallel to the pattern-projector, meeting MN in C,C′.

Then, since BC, B′C′ are parallel to the pattern-projector, they are parallel to one another (*xi.* 9). Wherefore the two lines, AB, BC, are respectively parallel to the two lines A′B′, B′C′, and, therefore, the planes ABC, A′B′C′ are parallel (*xi.* 15).

Again, since the parallel planes ABC, A′B′C′ are cut by the plane MN, their sections, AC, A′C′, with it are parallel (*xi.* 16); and these are the projections of AB, A′B′ upon the plane MN by projectors parallel to a given pattern-projector. Whence, &c.

The proposition as here enunciated is somewhat more general than we require just now: but as the demonstration is precisely similar to that more limited case of orthographic projection, I have put it down in the general form.

IV. It follows from the preceding investigation, that if the projections of two lines upon both the horizontal and vertical planes be parallel, they are the projections of parallel lines. Or, in reference to the annexed figure, where $\alpha_1\beta_1$, $\alpha_2\beta_2$ are the projections of a line continued through the whole visible region, and $\gamma_1\delta_1$, $\gamma_2\delta_2$ parallel respectively to the former projections are the projections of another line, the two lines projected will be parallel. Fig. 6.

It also follows, that if the projections a_1, a_2 of a point be given, and likewise those of a line $\alpha_1\beta_1$, $\alpha_2\beta_2$, then if it be required to draw the projections of a line through the given point which is parallel to the given line, we have only to draw through a_1 the line $\gamma_1\delta_1$ parallel to $\alpha_1\beta_1$ in the horizontal plane, and through a_2 the line $\gamma_2\delta_2$ parallel to $\alpha_2\beta_2$ in the vertical plane. These will be the projections required; and the traces of the line, γ_1, δ_2 will be determined as already explained in II.

Having disposed of these preliminary matters (which certainly ought to be amongst the *most ordinary topics of professional education*, and I trust the day is not distant when they will be so), I proceed to the main purpose of this letter, viz.

V. Fig. 7.—*The projection of Architectural Shadows.*

1. *For Points.*—Let OX be the ground-line, R_1, R_2 the horizontal and vertical projections of the pattern-ray, a_1, a_2 the projections of a point, and b_1, b_2 those of a second point.

Through a_1 a_2 draw the projections of a line parallel to R_1, R_2, and find its traces a_1, a_2. Now the visible one of these traces is a_2 on the *vertical plane*, and is the shadow of the given point, whose projections are a_1, a_2.